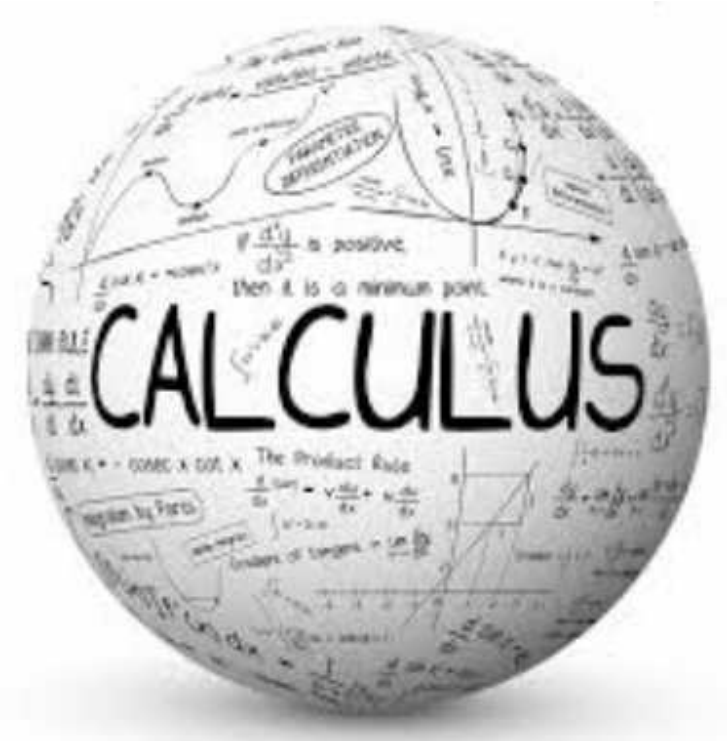
Name _____

2020 AP Calculus BC Summer Project



FOR MORE INFORMATION, please contact:

Mr. Sean Powers

Director of Science, Technology, Engineering, and Mathematics (STEM)

spowers@carteretschools.org **732-541-8960, ext 6032**

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Directions : Please answer these questions in a well-organized notebook. Please start a new problem on a new page.

- Unless otherwise directed, answer to three decimal places.
- Show all work.
- A graphing Calculator may be required for some problems. In these cases, describe the input. You can access the online graphing calculator at the link given below. . You will need to start the test for Algebra II. The calculator icon will be at the top of the screen. You do not log into the test.

https://nj.testnav.com/client/index.html#login?username=LGN688322866&password=3SJ3PDKX

This packet is due during the first week of school. Please email me at lmasse@carteretschools.org if you have questions or comments. I have established a Google Classroom where we can "meet" over the summer and discuss the project.

Calculus BC Summer Project Fun - 2020
Class code: qwxa2we

Have a good summer.

Dr. L. Masse

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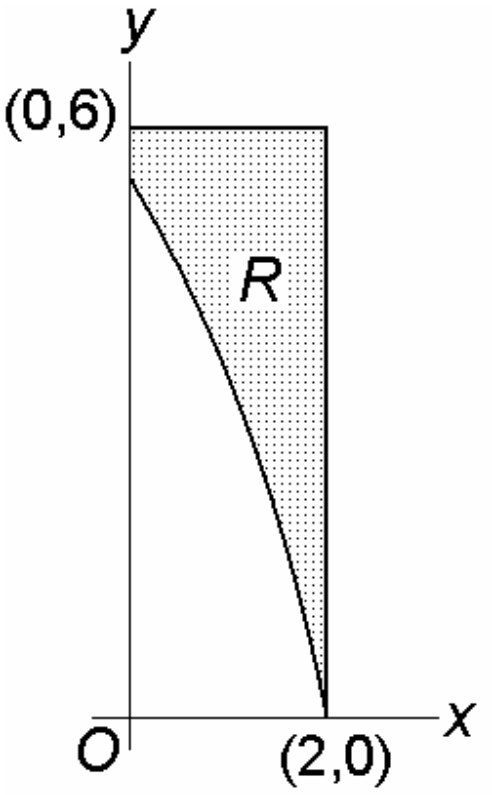
1. In the figure below, the region *R* is bounded by the *y*-axis, the graph of $y = 5\ln(3-x)$, and the lines at $y = 6$ and $x = 2$.



(a) Find the area of region *R*.

(b) Find the volume of the solid that is generated by revolving the region *R* about the line $y = 9$.

(c) Find the volume of the solid with region *R* as its base and whose cross-sections perpendicular to the x-axis are semicircles.

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2. The horizontal cross-section of a stalagmite at a certain height is always perfect circle. At time t, measured in years, the radius of this circle is $r(t)$, measured in centimeters. We know that $\frac{dr}{dt} = \frac{1}{160}\left(3 + \sin\left(t^2\right)\right)$ and that $r(0) = 9$.

(a) Write an expression for $r(t)$ that involves an integral. Use this expression to evaluate $r(4)$.

(b) Let $A(t)$ be the area of the circular cross-section at time t. Find $\frac{dA}{dt}$ at time $t = 2$.

(c) Evaluate $\int_1^4 A'(t)\,dt$. What is the physical interpretation of this definite integral?

3. The functions f and g are defined by $f(x) = \int_0^{2x} \sqrt{9 + t^2}\, dt$ and $g(x) = f(\cos x)$.

(a) Find $f'(x)$ and $g'(x)$.

(b) Write an equation for the tangent line approximation to the graph of $y = g(x)$ at $x = \frac{\pi}{2}$.

(c) Write and evaluate an expression involving an integral which can be used to find the maximum value of g on the interval $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$.

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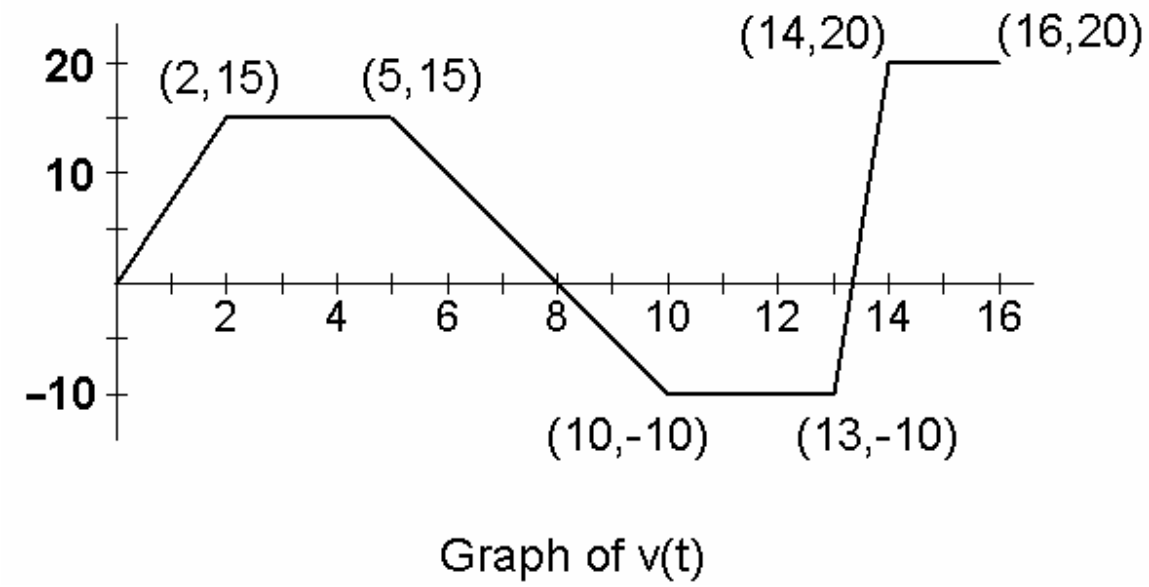
4. A fish is swimming between pond A and pond B along a perfectly straight stream. At time t = 0, the fish leaves pond A and enters the stream. For $0 \le t \le 16$, the velocity of the fish is modeled by the function graphed below.



Graph of v(t)

(a) When during the time interval $0 \le t \le 16$ does the fish change direction? Justify your answer.

(b) When during the time interval $0 \le t \le 16$ is the fish furthest from pond A? How far is the fish from pond A at this time? Justify your answer.

(c) What is the total distance that the fish swims during the time interval $0 \le t \le 16$? Justify your answer.

(d) Write expressions for the fish's acceleration, velocity and distance from pond A that are valid for $5 < t < 10$. Label your answers $a(t)$, $v(t)$, and $s(t)$.

5. Consider the following differential equation:

$$\frac{dy}{dx} = (y-1)^2 \cos(\pi x)$$

(a) Find the particular solution through the ordered pair (1,0).

(b) Is the solution in part (a) concave up, concave down, or neither at x = 1? Justify your answer.

6. Two objects are restricted to travel along the *x*-axis. During the time interval $0 \le t \le 6,$ the position function for object C is $c(t) = 3\cos\left(\frac{\pi}{4}t\right)$ and the position function for object D is $d(t) = 2t^3 - 21t^2 + 60t + 1.$

(a) When, during the time interval $0 \le t \le 6,$ is object C traveling to the right? Justify your answer.

(b) When, during the time interval $0 \le t \le 6,$ are the two objects travelling in opposite directions? Justify your answer.

(c) Find the acceleration at time $t = 3$ of object C. Is object C speeding up, slowing down, or neither at time $t = 3$? Justify your answer.

(d) Write an expression for the average distance between the two objects over the time interval $1 \le t \le 4.$ What is this average distance?